1. (7 pts) Find the absolute minimum and absolute maximum values of the function $g(u) = u^3 - 9u^2 + 15u + 7$ in the interval $[0, 8]$.

Step 1. *Find critical point(s) ...*

$$g'(u) = 3u^2 - 18u + 15 = 3(u^2 - 6u + 5) = 3(u-1)(u-5); \quad g'(u) = 0 \implies u_1 = 1 \text{ or } u_2 = 5$$

Observation: *Both critical points lie in the interval* $[0, 8]$.

Step 2. *Evaluate $g(u)$ at the endpoints of the interval and at the critical points in the interval—the largest value will be the maximum and the smallest value will be the minimum...*

u	$g(u)$	
0	7	
1	14	
5	-18	$\Longleftarrow$ absolute minimum in $[0, 8]$
8	63	$\Longleftarrow$ absolute maximum in $[0, 8]$

2. (6 pts) Find the ***absolute minimum*** value of the function $h(t) = 0.05t + 17 + \dfrac{200}{t}$ in the interval $(0, \infty)$. *Justify your claim* that the value you found is the absolute minimum.

Step 1. *Find critical point(s) ...*

$$h'(t) = 0.05 - 200t^{-2}; \quad h'(t) = 0 \implies 200t^{-2} = 0.05 \implies t^2 = \frac{200}{0.05} = 4000 \implies t = \pm\sqrt{4000}.$$

Observation: *There is only one critical point in the interval* $(0, \infty)$: $t^* = \sqrt{4000} \approx 63.25$.

Step 2. *Classify the critical value using the first derivative test* <u>*or*</u> *the second derivative test...*

First derivative test: $h'(1) = 0.05 - 200 = -199.95 < 0$ *and* $h'(100) = 0.05 - \frac{200}{100^2} = 0.03 > 0$, *so* $h(\sqrt{4000}) \approx 23.325$ *is a* ***relative minimum*** *value.*

Second derivative test: $h''(t) = 400t^{-3}$ *and* $h''(\sqrt{4000}) = 200 \times 4000^{-3/2} \approx 0.0008 > 0$, *so* $h(\sqrt{4000}) \approx 23.325$ *is a* ***relative minimum*** *value.*

Step 3. *Conclusion:* $h(\sqrt{4000}) \approx 23.325$ *is a* ***relative*** *minimum value and* $t^* = \sqrt{400}$ *is the* ***only critical point*** *in* $(0, \infty)$, *so* $h(\sqrt{4000}) \approx 23.325$ *is the* ***absolute minimum value*** *in in* $(0, \infty)$.

3. (7 pts) Find the critical point(s) and critical value(s) of the function $f(x) = 5x^2 e^{-0.25x}$ and use the *first derivative test* to classify the critical value(s) as relative minima, relative maxima or neither.

Step 1. *Find the critical point(s)...*

$$f'(x) = 10xe^{-0.25x} + 5x^2 e^{-0.25x} \cdot (-0.25) = 5xe^{-0.25x}(2 - 0.25x); \quad f'(x) = 0 \implies x_1 = 0 \text{ or } x_2 = 8.$$

Step 2. *First derivative test...*

$f'(-1) = -5 \cdot e^{0.25} \cdot 2.25 < 0$ *and* $f'(1) = 5 \cdot e^{-0.25} \cdot 1.75 > 0$, *so* $f(0) = 0$ *is a* ***relative minimum value***.[†]

$f'(1) = 5 \cdot e^{-0.25} \cdot 1.75 > 0$ *and* $f'(12) = 60e^{-3}(2-3) < 0$, *so* $f(8) = 320e^{-2} \approx 43.3073$ *is a* ***relative maximum value***.[‡]

[†] Actually, $f(0) = 0$ is the absolute minimum value in the interval $(-\infty, \infty)$... Why?

[‡] Additionally, $f(8)$ is the absolute maximum value in the interval $(0, \infty)$... Why?