NAME: ____

1. Find the derivatives (using the formulas, not the definition).
Clean up your answers.

(a) (3 pts) $y = 5x^2 \ln(x^3+1)$; $\dfrac{dy}{dx} = 10x\ln(x^3+1) + 5x^2 \cdot \dfrac{3x^2}{x^3+1} = 10x\ln(x^3+1) + \dfrac{15x^4}{x^3+1}$

(b) (3 pts) $f(t) = 10e^{5-2t}$; $f'(t) = 10e^{5-2t}\cdot(-2) = -20e^{5-2t}$

2. (6 pts) Find the ***degree*** 2 Taylor polynomial for the function $f(x) = \sqrt{x}$ centered at $x_0 = 4$, and use this polynomial to estimate $\sqrt{4.4}$.

(a) $f(4) = \sqrt{4} = 2,\ f'(x) = \frac{1}{2}x^{-1/2} \implies f'(4) = \frac{1}{2}4^{-1/2} = \frac{1}{4},$

$f''(x) = -\frac{1}{4}x^{-3/2} \implies f''(4) = -\frac{1}{4}4^{-3/2} = -\frac{1}{32}.$

(b) Quadratic (degree 2) Taylor polynomial for $f(x)$ centered at $x = 4$:

$$T_2(x) = f(4) + f'(4)(x-4) + \frac{f''(4)}{2}(x-4)^2 = 2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2$$

(c) Approximating $\sqrt{4.4}$:

$$\sqrt{4.4} \approx T_2(4.4) = 2 + \frac{1}{4}(4.4-4) - \frac{1}{64}(4.4-4)^2 = 2 + 0.1 - 0.0025 = 2.0975.$$

3. The demand equation for a monopolist's product is $q = 15\sqrt{240-1.4p}$

(a) (4 pts) Find the price-elasticity of demand for this good ***as a function of the price*** p.

(b) (2 pt) What is the price elasticity of demand when $p = 100$?

(c) (2 pts) Use your answer to (b) to find the firm's *marginal revenue* when $p = 100$.

Show/Explain your work.

(a)

$$\eta = \frac{dq}{dp}\cdot\frac{p}{q} = \overbrace{\cancel{15}\cdot\frac{1}{2}(240-1.4p)^{-1/2}\cdot(-1.4)}^{dq/dp}\cdot\frac{p}{\cancel{15}\sqrt{240-1.4p}}$$
$$= -\frac{0.7}{\sqrt{240-1.4p}}\cdot\frac{p}{\sqrt{240-1.4p}}$$
$$= -\frac{0.7p}{240-1.4p}$$

(b) $\eta\Big|_{p=100} = -\dfrac{0.7p}{240-1.4p}\Big|_{p=100} = -\dfrac{70}{100} = -0.7$

(c) $\dfrac{dr}{dq}\Big|_{p=100} = p\left(1+\dfrac{1}{\eta}\right)\Big|_{p=100} = 100\left(1+\dfrac{1}{-0.7}\right) = -\dfrac{300}{7} \approx -42.86$