1. Use the *differentiation formulas* to find the derivatives of the functions below.

Clean up your answers.

(a) (3 pts) $f(x) = 2 \cdot \sqrt[3]{x} - \dfrac{3}{x^2} = 2x^{1/3} - 3x^{-2}$; $\qquad f'(x) = 2 \cdot \dfrac{1}{3}x^{-2/3} - 3(-2)x^{-3} = \dfrac{2}{3}x^{-2/3} + 6x^{-3}$

(b) (3 pts) $v = (3u^2+1)(u^4 - 3u + 2)$;

$$\begin{aligned}\frac{dv}{du} &= 6u(u^4 - 3u + 2) + (3u^2+1)(4u^3 - 3) \\ &= 6u^5 - 18u^2 + 12u + 12u^5 + 4u^3 - 9u^2 - 3 \\ &= 18u^5 + 4u^3 - 27u^2 + 12u - 3\end{aligned}$$

(c) (3 pts) $y = \dfrac{3x^2+1}{4x+5}$; $\qquad y' = \dfrac{6x(4x+5) - 4(3x^2+1)}{(4x+5)^2} = \dfrac{12x^2 + 30x - 4}{(4x+5)^2}$

2. (5 pts) Use the ***definition*** of the derivative to find the slope of the curve $y = \dfrac{2}{x}$ at the point $x_0 = 1$.

Show your work and use the <u>limit</u> notation/properties correctly.

$$\left.\frac{dy}{dx}\right|_{x=1} = \lim_{h \to 0} \frac{\dfrac{2}{1+h} - \dfrac{2}{1}}{h} = \lim_{h \to 0} \frac{\dfrac{2}{1+h} - \dfrac{2(1+h)}{1+h}}{h} = \lim_{h \to 0} \frac{-2\cancel{h}}{\cancel{h}(1+h)} = \lim_{h \to 0} \frac{-2}{1+h} = -2.$$

3. (6 pts) A firm's ***marginal revenue*** function is given by

$$\frac{dr}{dq} = 7 - \sqrt{4q^2+9},$$

where revenue (r) is measured in \$1000s per month and output (q) is measured in 100s of widgets per month. By approximately how much will the firm's monthly revenue increase if its monthly output increases from 200 widgets to 260 widgets?

Show your work, pay attention to the units and express your answer in dollars.

(1) Output is measured in 100s of units, so a starting output of 200 units means that $q_0 = 2$, and an increase to 260 units means that $\Delta q = 0.6$.

(2) To estimate the change in revenue, we use linear approximation:

$$\Delta r \approx \left.\frac{dr}{dq}\right|_{q=2} \cdot \Delta q = \left.\left(7 - \sqrt{4q^2+9}\right)\right|_{q=2} \cdot \Delta q = \left(7 - \sqrt{16+9}\right) \cdot (0.6) = 1.2$$

In words: the firm's revenue will increase by about \$1200.00 (because revenue is measured in \$1000 s).